WESTERGAARD'S METHOD OF EXPECTED DEATHS AS APPLIED TO THE STUDY OF INFANT MORTALITY

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A characteristic of infant mortality data, as well as of many other kinds of economic and social material, is that many and varied causes are at work to produce the given results. A prime requisite in a method for analyzing such data, therefore, is that it shall permit of separating out and determining the influence of each factor when other conditions or factors are held constant. This requisite is possessed by Professor Westergaard's method of expected deaths used in conjunction with multiple classification. The purpose of this paper is to present to American readers the mode of applying that method to infant mortality data, and incidentally to give certain partial results of applying it to data gathered by the United States Children's Bureau. It may be remarked in passing that the method may be applied as effectively to many other problems.

Among the causal factors with which we are concerned in analyzing infant mortality data the following may be included, at least tentatively: sex, age, single or plural birth, type of feeding, care of mother during pregnancy, mother's employment, care of infant during first year of life, economic status of the family, age of mother, order of birth, interval from the preceding birth, pregnancy of the mother following the the birth, nationality of the mother, death of the mother, housing conditions, and illegitimacy.

In relation to each factor three questions are always pertinent—the questions of relative mortality, of prevalence, and of the contribution which the factor makes to the general result. Thus, with respect to single and plural births, the first question is whether twins and triplets have a greater mortality than single births; the second, what proportion of the total births are plural; and the third, do twins and triplets contribute a large quota to the infant deaths? The answer to the last question obviously depends upon the answers to the other two. The first and third questions are often confused, as in the inquiry: Is the mother's employment an important factor in infant mortality? This is capable of two interpretations, one—of greater importance in an analysis of causes—relating to relative mortality, *i. e.*, whether infants whose mothers are employed are subject to a higher rate of mortality than infants whose mothers are not employed, and the other

relating to the amount of contribution to the total mortality, *i. e.*, whether any excess mortality among infants of employed mothers in connection with a large or small prevalence of mothers' employment results in an important contribution to infant mortality. This question is of secondary importance since the contribution is the resultant of two primary elements. The method of analysis should distinguish between these primary and secondary elements in the problem.

The conclusions reached with regard to the relative mortality associated with each factor, so far as the factor itself is conceived to represent a true causal element, are tentative; they are subject to qualification if intercorrelations with other factors are present. It is therefore important to study the intercorrelations of the several factors. These intercorrelations have two aspects: first, whether the presence of one factor is associated or correlated with the presence of another, and second, whether the relative mortality exhibited by the first factor varies as it is associated with the second. The amount of qualification of the tentative conclusion is dependent upon the closeness of correlation between the factors, the importance of the associated factors, and the degree of excess mortality connected with them. So far as no such intercorrelations are present, the tentative conclusion remains unmodified.

These principles may be apprehended more clearly in illustrations. An analysis of a group of births by sex shows (1) that males have a slight preponderance over females, and (2) that the mortality rate among males in the first year of life is slightly higher than that among females. To determine whether this tentative conclusion (2) requires qualification, the prevalence of male births must be studied in reference to other factors, types of feeding, single and plural births, age of mother, order of birth, earnings of father, employment of mother, etc.; if the proportion of male births does not vary with these other factors, the conclusion may be accepted as expressing the true excess mortality among male births.

On the other hand, an analysis of a group of births into single and plural births shows (1) that about 2 per cent of all births are twins or triplets, and (2) that the mortality among twins and triplets is between three and four times that among single births. The prevalence of twins and triplets is then studied with regard to other factors, age of mother, order of birth, type of feeding, earnings of father, employment of mother, etc. If the analysis indicates that twins and triplets are slightly more prevalent among births to older mothers and among births of later orders, and that they are more likely to be artificially fed, the tentative conclusion as to relative excess mortality among

twins and triplets over that among single births is subject to modification. The amount of modification arising from its correlation with artificial feeding, for example, will be greater the larger the proportion of twins and triplets artificially fed and the greater the excess mortality among artificially fed over that among breast-fed infants.[1] But with regard to each factor with which the proportion of twins and triplets is associated, the question arises whether the higher mortality among artificially fed, among the births to older mothers, and among the births of late orders is wholly or in part determined or influenced by the greater prevalence of twins and triplets.[2] The question, therefore, of the degree of modification of the tentative conclusions can be determined only after analysis.

The first recourse in analysis is usually to that universally applicable method, simple classification, or, as it may be termed when several factors are included, multiple classification. If births and deaths are classified each by the same factors, such as earnings of father or nationality of mother, the rates of mortality indicate whether the factors used in the classification can be considered as having an influence upon infant mortality. If two factors such as the two cited are correlated, the births and the deaths must be subclassified; the rates by earnings of father in each nationality group indicate whether earnings of father can be considered a factor independent of nationality, and the rates by nationality of mother in each earnings group indicate whether nationality of mother can be considered a factor independent of earnings. Evidently by this mode of comparison the effect of one factor upon the rate can be studied by the simple device of studying its influence in a group within which another factor does not vary. Multiple classification, if carried far enough, possesses the essential requirements for isolating the effect of a single factor.[3] But as more and more classifications are introduced, on the one hand the results become increasingly difficult for the mind to grasp, and on the other, the material becomes more and more subdivided until, finally, the base

[1] The degree of excess mortality of twins and triplets over that among single births may itself vary with the type of feeding.

[2] In this particular illustration the influence of the greater prevalence of twins and triplets among the artificially fed upon the relative mortality among artificially as compared with breast-fed infants is not great.

[3] In mathematical terms, assuming that all causal factors can be expressed numerically, the infant mortality rate, X_1, can be stated in terms of its several independent causal factors: $X_1 = f\ (X_2, X_3, X_4\ .\ .\ .\ .\ X_n)$ where X_2, X_3, etc., are the independent causal factors. The partial derivative of X_1 with respect to any one factor, for example, $\frac{\delta X_1}{\delta X_2}$, measures the change in X_1 due to the change in X_2; in other words, measures the influence of X_2 upon X_1 when all other variables are considered constant. Geometrically, this partial derivative is the slope of X_1 measured along the axis of X_2. In a classification table, for example, Table V, values of X_1 are given for different values of X_2, with other factors, so far as they are included in the classification table, constant.

becomes too small to ensure valid conclusions. To meet the first of these difficulties, recourse may be had to the familiar process of selection of data. For example, the influence of varying prevalence of plural births may be eliminated by confining the analysis to single births, and the influence of differences in types of feeding may be eliminated by further omitting all except breast-fed infants. But to obviate the difficulty that the numbers in the final group may not be large enough to ensure valid results, recourse must be had to some method of summation.

To meet these difficulties in the use of the method of multiple classification recourse may be had to the method of expected deaths developed by Professor Harald Westergaard.[1] This method, in fact, offers a simple and easily applicable way of isolating the influence of a single factor from that of other associated factors, and of stating the results in clear and definite terms.

Because the wide applicability of the Westergaard method seems not to be generally appreciated in this country, it will be described briefly before returning to the subject of its application to a specific infant mortality problem. Though designated as the "method of expected deaths" on account of its first application to mortality data, it is not restricted in its application to these problems but can be applied to a great many other fields, and in these fields it might be termed the method of "expected cases," or, generally, "expected results." The essence of the method lies in so calculating the expected numbers that a comparison between them and the actual numbers will be free from the disturbing influence of one, two, or several factors. By this means the effect of a single cause is isolated from the influences of other causes.

The best known, as well as the earliest, applications of the method were to the problems of eliminating variations in age and sex composition in comparisons of mortality rates in urban and rural populations and of eliminating variations in age composition in comparisons of occupational mortality.[2] Since, in these applications, identical or

[1] *Die Lehre von der Mortalität und der Morbilität: Anthropologisch Statistiche Untersuchungen.* Jena. G. Fisher, 1882, pp. 28–30; and "Scope and Method of Statistics," this JOURNAL, Vol. XV, No. 115, Sept., 1916, pp. 260–64.

[2] The method of standardizing or correcting death rates for differences in age composition was clearly set forth in the introductory chapter of *Die Lehre von der Mortalität*, pp. 28–30. In England, in the Registrar-General's Report for 1882, this method of correcting for varying age composition in comparison of urban and rural districts was used. (Forty-fifth Annual Report of the Registrar-General of Births, Marriages, and Deaths in England, Abstracts of 1882, C–4009 London, 1884.) The use of corrected death rates for Hamburg by Koch in 1883 is mentioned by Von Mayr, *Statistik und Gesellschaftslehre*, Vol. II, p. 219. Westergaard himself mentions the use of the method of expected deaths by Ratcliffe, in a first report, *Observations on the Rate of Mortality and Sickness amongst Friendly Societies*, Manchester, 1850, quoted in *Die Lehre von der Mortalität*, p. 287. Ratcliffe, however, seems not to have appreciated its importance, for in a second report the method was apparently not used.

"standard" rates were applied to varying populations, or varying rates to identical or "standard" populations, the method is perhaps better known to English and American statisticians as the method of standards. But there is this difference: the name "method of standards," by suggesting that any standard is equally good, has sometimes led to the use of a standard that was not adapted to the particular problem; while the "method of expected cases," by comparing expected with actual numbers, always keeps as close as possible to the actual problem under consideration.[1]

With this preliminary statement, we may proceed to a study of the actual operation of the method in which its merits and its limitations will be revealed much more clearly than by mere description. The first question chosen to illustrate the operation of this method of expected deaths is that of determining the influence of order of birth upon infant mortality.[2] Order of birth is closely associated with age of mother, since a large proportion of the births of later orders are to the older mothers; it is also correlated to some extent with the prevalence of twins and triplets; and it is correlated slightly with low income, since larger families as measured by the number of births are more prevalent at the lower income levels.

TABLE I

AGE OF MOTHER AND ORDER OF BIRTH; LIVE BIRTHS IN EIGHT CITIES

Order of birth	Live births							
	Total	Age of mother						
		Under 20	20–24	25–29	30–34	35–39	40 and over	Not reported
Total	22,967	1,584	6,879	6,618	4,231	2,688	958	9
First	6,230	1,227	3,125	1,349	399	114	14	2
Second	4,954	311	2,196	1,621	599	204	22	1
Third	3,328	39	968	1,407	609	260	45	.
Fourth	2,481	5	412	1,043	694	268	57	2
Fifth	1,767	2	133	624	631	306	69	2
Sixth	1,263	....	32	339	480	326	85	1
Seventh	921	....	9	138	353	320	101	.
Eighth	677	....	3	68	226	289	91	.
Ninth	470	....	1	20	126	224	98	1
Tenth and later	876	....	....	9	114	377	376	.

Table I presents a distribution of live births classified by age of mother and order of birth.[3] An inspection of the distribution shows

[1] Compare the recent discussion of the best methods of choosing weights in the problem of measuring price increases—one of the familiar examples of the method of standards.

[2] For a similar application of the method to this problem see Westergaard, *op. cit.*, Revised ed., 1901, pp. 371–75.

[3] The material is for live births in selected years in eight cities studied by the Children's Bureau, and is taken from a report on infant mortality shortly to be issued.

at once that the lower orders of birth are associated with the younger ages and the higher orders with the higher ages. The infant mortality rates for the order of birth groups beginning with first births are 104.7, 95.7, 104.6, 108.8, 118.8, 122.7, 136.8, 135.9, 146.8, and for tenth and over, 181.5. It may be noted that the rate for first births is higher than that for second, and that there is a rapid increase in rate from 95.7 for second to 181.5 for births tenth and later in order. The average rate for births of all orders is 111.2; expressing this average as 100, the following numbers show the variation in rates by order: 94.2, 86.1, 94.1, 97.8, 106.8, 110.3, 123.0, 122.2, 132.0, 163.2. The rates for age-of-mother groups are: Under 20, 135.7; 20–24, 109.3; 25–29, 101.4; 30–34, 104.7; 35–39, 126.5; and 40 and over, 136.7.

In view of the fact that the infant mortality rates among births to older mothers are high and that these are largely births of the later orders, the question arises whether the age of mother is the factor that produces the high rate among the births of later orders, whether the order of birth is itself the causal element, or whether both factors contribute to the result.

This question can be answered by ascertaining what the rates by order of birth would have been on the assumption that age of mother is the sole causal factor, and comparing the actual with these calculated rates. Applying the average rates by age of mother to the distribution of births by order of birth and age of mother, the deaths in each sub-group can be calculated; the deaths in each order-of-birth group can then be added, and these totals compared with the actual deaths that occurred. If order of birth has no causal influence aside from its correlation with age of mother, the number of actual and of expected deaths will be equal. If, however, the actual deaths do not equal those

TABLE II

RELATIVE MORTALITY BY ORDER OF BIRTH, WHEN INFLUENCE OF AGE OF MOTHER IS ELIMINATED; LIVE BIRTHS IN EIGHT CITIES

Order of birth	Deaths			Ratio of original rate to average
	Actual	Expected	Ratio of actual to expected	
Total	2,555	2,555.3	100.0	100.0
First	652	704.1	92.7	94.2
Second	474	538.6	88.0	86.1
Third	348	356.8	97.6	94.1
Fourth	270	266.5	101.2	97.8
Fifth	210	192.7	109.0	106.8
Sixth	155	141.2	109.7	110.3
Seventh	126	106.2	118.4	123.0
Eighth	92	79.9	115.2	122.2
Ninth	69	57.3	120.2	132.0
Tenth and later	159	112.0	142.0	163.2

expected, the influence of order of birth or of other factors associated with it is revealed in the proportion by which the actual deaths fall short of or exceed the expected deaths. The figures in Table II show the actual deaths in comparison with deaths when the influence of age of mother is eliminated.

The explanation of this calculation can be phrased in other terms. The series of ratios show, in fact, the influence of order of birth (together with associated factors) when the effect of the disproportionate weighting of the various groups with births to younger or to older mothers is eliminated, or, more simply, when the influence of age of mother is eliminated from the relative rates.

The effect of eliminating the influence of age of mother appears slightly to diminish the excess mortality of first as compared with second births, and to lessen the excess mortality of births of later as compared with births of earlier orders. The influence of order of birth, though diminished, is still marked.

Table III presents a similar computation showing the influence of order of birth after additional factors are eliminated, to wit, earnings of father and plurality of birth. The computations made in constructing this table are much more complicated than those made in constructing the preceding one, since each group of births classified by earnings of father, order of birth, and age of mother is multiplied by the average rate in the corresponding earnings-of-father and age-of-mother group; the results are then totaled to give the expected deaths by order of birth. These expected deaths are the numbers that would be expected if full weight were given to the factors age of mother and earnings of father, but if no influence of order of birth were present. The differ-

TABLE III

RELATIVE MORTALITY BY ORDER OF BIRTH, WHEN INFLUENCES OF AGE OF MOTHER AND EARNINGS OF FATHER ARE ELIMINATED; SINGLE LIVE BIRTHS IN SEVEN CITIES

Order of birth	Deaths		
	Actual	Expected	Ratio of actual to expected
Total	2,196	2,196.0	100.0
First	586	589.8	99.4
Second	410	454.3	90.2
Third	297	302.3	98.2
Fourth	232	232.9	99.6
Fifth	175	168.7	103.7
Sixth	119	124.0	96.0
Seventh	108	95.9	112.6
Eighth	75	71.6	104.7
Ninth	61	54.5	111.9
Tenth and later	133	101.8	130.6

ences between the actual and expected deaths show, as before, the influence of order of birth after the disturbing effects of these other factors have been eliminated.

It is of interest to note that the general trend of the rate by order of birth remains unaltered; the first births have a slightly higher rate than the second, and from the second there is a general tendency to increase with the later orders. The elimination of the two additional factors—earnings of father and plural birth—tends, however, to reduce still further the influence of order of birth as compared with that indicated by the original averages.

A second question concerns the influence of age of mother. Table IV presents the relative ratios between the average rate and the rates

TABLE IV

RELATIVE MORTALITY BY AGE OF MOTHER, WHEN INFLUENCE OF CERTAIN OTHER FACTORS IS ELIMINATED; LIVE BIRTHS IN EIGHT CITIES

Age of mother	Ratio of actual to expected deaths		
	On basis of average rate	Eliminating influence of order of birth	Eliminating influence of order of birth, earnings of father, and plural births
Total	100.0	100.0	100.0
Under 20	121.2	131.8	121.3
20–24	98.5	106.8	101.9
25–29	91.2	95.2	95.5
30–34	94.2	90.4	93.3
35–39	113.8	97.5	105.3
40 and over	122.9	92.4	91.1

for each age-of-mother group, first with no eliminations, second, when the influence of order of birth is eliminated, and third, when the influences of order of birth, earnings of father, and plural births are eliminated.

The effect of eliminating the influence of order of birth appears in the removal of the high mortality rate among the older mothers, which appears to be due largely to the influence of the high mortality rate attending births of late orders. The effect of eliminating the influence of earnings of father appears in lessening the excess mortality among the young mothers, a large proportion of whom are in families with low earnings.

Since the primary purpose of this paper, as stated above, is to illustrate the application of Professor Westergaard's method of isolating the influence of the several factors, these examples may perhaps suffice. They are not intended to suggest that the analysis made is sufficient, but are presented to show how the method can be applied to eliminate

one, two, or more factors. As many factors should be eliminated as a thorough consideration of the problem shows to be necessary. In many cases, however, the limitations in the amount or in the character of material available do not allow the exhaustive analysis that the nature of the problem requires.

One suggestion might be made as to an appropriate notation to express the operations and results of multiple classification and of the method of expected deaths. Letting 1 denote the infant mortality rate, 2 order of birth, 3 age of mother, 4 earnings of father, 5 nationality of mother, 6 plural births, and similar subscripts for other factors, then Table I, for example, may be designated briefly B 2.3, or births classified by age of mother and order of birth. If subclassification by nationality of mother is introduced the table becomes B 2.3–5. Letting R denote the series of ratios of actual and expected deaths that shows the influence of a given factor such as order of birth upon the infant mortality rate, then the relation between the rate and order of birth may be stated simply $R_{1.2}$; if a factor such as age of mother is eliminated, it may be indicated by placing the corresponding number after a colon following the numbers indicating the relation shown, thus $R_{1.2:3}$. The third column of ratios in Table IV, for example, may be designated, $R_{1.3:2\text{-}5\text{-}6}$.

Among the limitations of the method considered in this paper may be mentioned the following points: (1)The method requires a considerable amount of computation if more than one or two factors are eliminated; (2) the application of the method requires a mass of material commensurate with the number of factors to be eliminated, though no more than would be required by any other equally satisfactory method; (3) since it is an averaging process the method will yield satisfactory results only when an average is appropriate.

The last point may be illustrated by Table V, which shows infant mortality rates classified by age of mother and order of birth. Among births to young mothers the rate of infant mortality rises sharply from first to third; while among births to mothers of the oldest groups shown the rate, high for first births, falls to a minimum for third births and rises to a very high figure for births tenth and later in order. These characteristic variations of the rate are obscured by an averaging process,[1] unless such a process takes into account the interval between births which also enters as a factor in this case. Again, the averaging

[1] In mathematical terms, the partial derivative which measures the contribution of X_2 to X_1 when other factors are held constant may in general contain constant terms in X_3, X_4, and other factors. The condition that the partial derivative shall not vary with X_3, X_4, etc., requires an averaging process to find an expression for $\frac{\delta X_1}{\delta X_2}$ that contains no terms in any other variable. Such a process is ap-

process indicated (Table IV) that age of mother was not an unfavorable element in the rate of mortality among infants of older mothers; but the classified table of rates shows that for first births the infant mortality rate is higher among births to older mothers than to mothers between 25 and 29 years of age.

A further difficulty arising from this averaging process is the question of the effect of the weights chosen. The results, as can easily be

TABLE V

INFANT MORTALITY RATES, BY AGE OF MOTHER AND ORDER OF BIRTH; LIVE BIRTHS IN EIGHT CITIES

Order of birth	Infant mortality rate [1]						
	Total	Age of mother					
		Under 20	20–24	25–29	30–34	35–39	40 and over
Total	111.2	135.7	109.3	101.4	104.7	126.5	136.7
First	104.7	125.5	100.5	90.4	102.8	131.6	.....
Second	95.7	167.2	105.6	74.0	76.8	112.7	.....
Third	104.6	.....	125.0	103.8	82.1	73.1	.....
Fourth	108.8	.....	155.3	99.7	106.6	82.1	.....
Fifth	118.8	.....	97.7	136.2	96.7	127.5	.....
Sixth	122.7	.....	.....	150.4	102.1	128.8	.....
Seventh	136.8	.....	.....	195.7	121.8	131.3	108.9
Eighth	135.9	.....	.....	.....	137.2	121.1	.....
Ninth	146.8	.....	.....	.....	174.6	133.9	.....
Tenth and later	181.5	.....	.....	.....	228.1	193.6	159.6

[1] Rates are not shown where base is less than 100.

shown, may vary considerably according to the weights. In this respect the method has the same advantages and the same weaknesses as the method of standardizing or correcting death rates, or of measuring price increases. Experience has indicated as a general rule to be followed that the weights chosen should agree as closely as possible with the "natural" weighting in the problem studied. Thus, if it is desired to eliminate the influence of age of mother, the actual weighting of the births by order of births and age of mother yields the most satisfactory results and introduces no new element of arbitrary weighting into the problem. Care must always be exercised, however, in applying the method to determine whether the results are affected by the weights. Errors of interpretation arising from these difficulties

propriate only when the types and values of the partial derivative do not vary materially for different values of other variables.

If the additional condition is imposed that the partial derivative with respect to each factor be a constant, the expression becomes linear and corresponds to the familiar partial straight line regression equation $X_1 = b_{12.34} \ . \ . \ . \ X_2 + b_{13.24} \ . \ . \ . \ X_3$, etc.

It is clear that the application of the Westergaard method is somewhat broader than that of the partial straight line regression equation, and that the latter is equally inapplicable to cases in which an averaging process obscures the true significance of the variations in the rates.

can always be avoided by a study of the rates classified by the several factors.

Among the advantages of the method are the following: (1) The results are stated in clear and simple terms; (2) the numbers upon which the results are based can be presented and the validity of the result judged thereby; (3) the method requires the constant use of the original material, which often suggests the necessity of taking into account factors, such as age of infant in connection with type of feeding, which might otherwise be overlooked; (4) it can be applied in certain cases even when the material cannot be classified so as to yield the full detail of rates; [1] (5) it is not restricted to linear relationships but indicates any type of average relationship which may exist; (6) it is not restricted to factors which can be expressed numerically but can be applied equally well to factors, such as nationality and employment of mother, which are not susceptible of statement in numerical terms; and (7) it places in true relation the actual effect of each contributing factor after the influences of other factors have been eliminated.

[1] For example, if the births can be fully but the deaths or stillbirths only partially classified. See Westergaard, *op. cit.*, Revised ed., pp. 333, 336; and "Scope and Method of Statistics," *loc. cit.*, p. 262.